Self-heating of a Thermistor in Temperature Measurements

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The electrical resistances of a thermistor have found wide application as thermometric parameters. For this purpose, suitable resistance bridges are used to measure the resistance or unbalanced voltages of the bridges due to the variation in resistance accompanying a change in the ambient temperatures to be measured. Usually, the sensitivity of the bridge is improved as the bridge voltage is increased; however, this improvement encounters self-heating or rises in the temperature of the resistance thermometer element due to the dissipation of the electrical power produced in the element. At first, there is difference in temperatures between that of the thermometer element and the ambient medium. Secondly, a considerable deviation from the simple $\exp(b/T)$ law for thermistors is anticipated. For instance, Hutchinson and White[1] showed that, above a certain value of bridge voltage, unsatisfactory results were obtained in calorimetry. Therefore, it is necessary to clarify the effect before proceeding to precise measurements in thermometry.

If we assume a thermal equilibrium between the heat due to electrical power dissipation, W, produced in a thermistor element and the thermal leakage, which is proportional to the difference between the temperature, T, of the thermistor element and the ambient temperatures T_0 (Newton's law of cooling): then

1) W. P. Hutchinson and A. G. White, *J. Sci. Instrum.*, **32**, 309 (1955).

$$h(T - T_0) = W = V^2/R \quad (1)$$

where h is the heat transfer coefficient between the thermistor element and the ambient medium, R is the electrical resistance of the thermistor, and V is the voltage across the thermistor. Further, we assume a usual $\exp(b/T)$ temperature-variation law of the resistance:

$$R = A\exp(B/T), \quad (2)$$

where A and B are experimentally-determined constants. Then, from Eq. (1), the temperature, T, of the thermistor element is given by the equation:

$$T = T_0 + (V^2/hR), \quad (3)$$

and the temperature differences, ΔT, between the element and the ambient medium, by:

$$\Delta T = T - T_0 = V^2/hR. \quad (4)$$

By substituting the T from (3) into (2), one obtains:

$$R = A \exp\left\{B\Big/\left(T_0 + \frac{V^2}{hR}\right)\right\}. \quad (5)$$

If the temperature rise, ΔT, of the thermistor element due to electrical-heat dissipation is small compared to T_0 and T ($\Delta T \ll T_0$, T), Eq. (5) can be approximated as follows:

$$R = R_0 - \frac{BV^2}{hT_0^2}, \quad (6)$$

where R_0 is a extrapolated resistance value as $V \to 0$ on the straight portion of the plots in Fig. 1, an explanation of which will be given later.

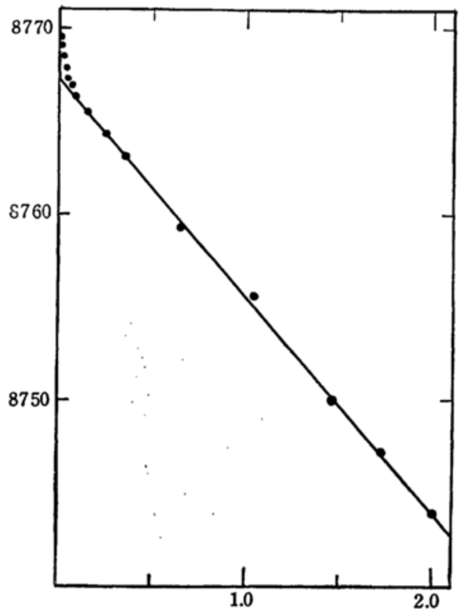

Fig. 1. Variation of electrical resistances with squares of voltage across thermistor at 25.00°C in water.

Experimental

Measurements were done on a bead-type OS41B-2 thermistor manufactured by Ōizumi Seisakusho, Ltd. The bead is coated with a skin film of glass to protect it. A d. c. Wheatstone bridge was used for the measurements of the resistance of the thermistor, and a National FM-5 primary cell was used for the power supply to the bridge. The thermistor was immersed in a thermostat held constant at ± 0.002°C for extended periods of time. The signal from the bridge was amplified by a d. c. current amplifier (Ōkura AMl001B), the output of which was used to derive a recorder (Yokogawa LER-12). The bridge voltage was varied and measured by a conventional d. c. voltmeter. The measurements were carried out at 25, 35, 40, and 45°C.

Results and Discussion

The results of the measurements are shown in Fig. 1. This figure shows that Eq. (6) is valid except in low voltages across the thermistor. The slope of the plot in Fig. 1, together with the measurements of the variation in the resistance of the thermistor with the temperature, allows the constants A, B, and h to be approximately obtained; the values are:

$$A = 3.76 \times 10^{-2}\,\Omega,$$
$$B = 3.69 \times 10^{3}\,{}^{\circ}\mathrm{K} \quad (7)$$

and

$$h = 3.54 \times 10^{-3}\,\mathrm{W}^{\circ}\mathrm{K}^{-1},$$

where A and B are evaluated by assuming a simple relation, $R_0 = A\exp(B/T)$, of the R values and h extrapolated from Eq. (6).

The effects of the variation in the voltage across the thermistor during temperature measurements are examined on this thermistor, the constants of which are given by (7) as follows:

1. The differences in temperature, ΔT, between that of the thermistor element and that of the ambient medium are evaluated from Eq. (4) as, for example, 7.05×10^{-3} ° ($V = 0.5$ V) and 3.22×10^{-2} ° ($V = 1.0$ V) on this thermistor.

2. The variation, ΔT, in the temperature of the thermistor due to the variation, ΔV, in voltage across the thermistor is given by:

$$\Delta T = (2V/hR)\Delta V \quad (8)$$

on the basis of Eq. (1). The variation in voltage must be maintained, if the variation in temperature should be kept less than ΔT, as follows:

$$(\Delta V/V) \times 100 < (hR/2V^2) \cdot \Delta T \times 100\%. \quad (9)$$

If the values of $\Delta T = 10^{-5}$° and $V = 0.5$ V are taken, Eq. (9) gives:

$$(\Delta V/V) \times 100 < 0.062\% \quad (10)$$

on the above-mentioned thermistor.

3. Let us now determine the errors in the temperature measurements caused by the self-heating of thermistor, if the simple $\exp(b/T)$ law of the thermistor is used. For this purpose, some numerical calculations were carried out on a model thermistor; in these calculations Eq. (5) was assumed to be valid, and the A, B and h constants were given by Eq. (7). Now let us see the temperature differences, δT, between the temperature, T°, obtained by using a simple $R=a\exp(b/T)$ law for this thermistor and that, T, given by Eqs. (5) and (7). The a and b constants can be determined by $R=a\exp(b/T)$ if the resistances are given as $R_1=10500\ \Omega$ and $R_2=7000\ \Omega$, and corresponding temperatures as T_1 and T_2 by Eq. (5), substituting the resistance values, R_1 and R_2. Then, using these values of a and b, the temperature, T°, is calculated for an arbitrary resistance, R, and for temperature differences, δT, between the T° and T values given by Eqs. (5) and (7) for the same resistance, R. Figure 2 shows the temperature differences, $\delta T=T^{\circ}-T$, thus calculated for various values of voltage, V, across the thermistor. Deviations from the true values of the temperature increase with the voltage across a thermistor, as shown in Fig. 2, if a thermistor whose calibration has been carried out at about 20 and 30°C using

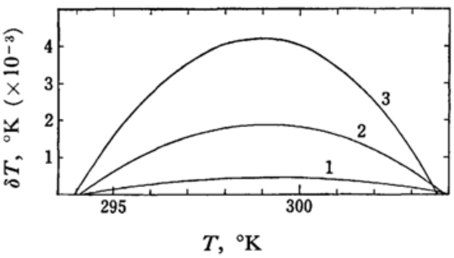

Fig. 2. Temperature differences $\delta T=T^{\circ}-T$. T are given by Eqs. (5) and (7). T° are calculated using relation $R=a\exp(b/T)$. Voltage V across the thermistor are $1V$ (1), $2V$ (2) and $3V$ (3).

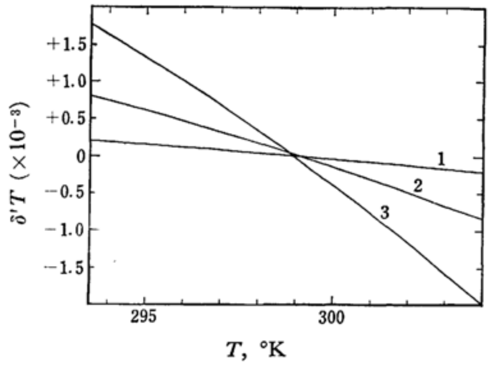

Fig. 3. First derivatives $\delta' T$ of δT with respect to T. $\delta' T$ are thought to be errors in measurements of temperature variations. Voltage V across the thermistor are $1V$ (1), $2V$ (2) and $3V$ (3).

$R=a\exp(b/T)$ is used to measure an absolute temperature between about 20 and 30°C.

A thermistor is generally used to measure temperature variations rather than absolute temperatures. Therefore, errors in the measurement of temperature variations should be examined. These errors may be defined as first derivatives, $\delta' T$, of δT with respect to T; they are shown in Fig. 3 for various voltages, V. Figures 2 and 3 also show an upper-limit value of voltage across the thermistor, a value which must not be exceeded for a desired degree of precision in the measurement of absolute temperatures and temperature variations when a simple $\exp(b/T)$ law is used.

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